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A method for optimizing urban rail transit timetable based on accurate power flow

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Abstract
The timetable of urban rail greatly affects its daily energy consumption. To improve the utilization of renewable energy between trains using timetabling has become an effective way to reduce energy consumption. Previous studies ignore or simplify the modelling of traction power supply network, which failed to accurately describe the flow of energy between trains through the power network. This paper proposed an optimisation method of energy efficiency timetabling considering the power flow of traction power supply network. First, an urban rail transit DC traction network model is established, and the current-vector iterative method is used to characterize the energy consumption. Then, a train timetable optimisation model is proposed to minimize the total energy consumption of the traction network system by adjusting the dwell time and section running time. The genetic algorithm is used to solve the optimisation problem. Finally, simulation result shows that the proposed method can accurately characterize the energy flow and effectively reduce the total energy consumption of the urban rail transit.

Keywords : Energy saving timetable, Genetic algorithm, DC traction network calculation, Multi-train optimisation

1. Introduction

Due to the large demand for commuting, several cities in China built many urban rail transit lines. The operation on railway systems, however, needs to consume plenty of electricity. For example, the national annual power consumption of urban rail transit in 2021 is 21.31 million kWh, exceeded 3‰ of national power consumption. In order to achieve SDGs provided by the United Nations, it is meaningful for energy-saving. Especially, half of the energy is consumed by traction of trains (Yang et al., 2016). And trains can convert kinetic energy into electricity and feed back to the traction network during regenerative braking (Wijaya et al., 2017). From this perspective, it is possible to save energy by optimising timetable (Takeuchi et al., 2019). Since there is no need to add equipment, timetable optimisation can significantly save operating costs on the basis of reducing energy consumption (Lv et al., 2019). Hence, it is a good optimisation method.

The timetable optimisation problem can be equivalent to synchronize the traction and regenerative braking for different trains. However, both the synchronized time and total energy consumption can be the optimisation objective function. In time synchronization, the genetic algorithm (GA) can be used to maximize the overlapping time (Li and Yang, 2020). When considering passengers' waiting time and system robustness additionally, the Pareto optimal solution can be found by NSGA-II (Yang et al., 2019). In energy consumption, besides GA (Li and Yang, 2020), the mixed integer linear programming can also be used (Jin et al., 2020). However, these calculation methods have the following common problems: First, the calculation examples are single section and single power supply, but the traction network in real urban rail transit systems are double power supply with parallel of upwards and downwards; Second, these researches use traction calculation to calculate energy consumption rather than traction network. Due to the non-linear of traction network, the results are not accurate enough. So, it is meaningful to calculate the energy consumption from the DC traction network, especially the output electricity from substations.

In order to determine the objective function of timetable optimisation and evaluate the optimisation results, it is necessary to model and solve the traction network of urban rail transit. Several researches explored the method to equivalent the train, substation and traction network (Cai et al., 1995a). In general, the equivalent model of train is power source. About the substation, due to its completive internal circuit, the Norton's theorem or Thevenin's theorem can be used to model it. The traction network can be equivalented by a ladder circuit (Ku and Liu, 2020), two-port network, or nodal admittance matrix (Mellitt et al., 1978). For another, several algorithms, such as current vector and Zollenkopf's bi-factorisation algorithm (Cai et al., 1995b), can be used to solve the power flow and energy consumption (Alnuman et al., 2018). To shorten the calculation time, this paper adopts the node admittance matrix to equivalent the traction network and adopts the current vector method to calculate.

Hence, this paper proposes a timetable optimisation method, which taking minimize the energy consumption as optimisation objective by improving the utilisation of regenerative braking energy. Based on the actual power flow and modelling of DC traction network, the actual substations electricity output can be calculated and to evaluate the optimisation effect.

This paper is structured as follows. In section 2, we give a model of DC traction network, and the solution of power flow calculation in power system. In section 3, the model of timetable energy-saving optimization is be established, and the method of improving genetic algorithm is explained. In section 4, the effectiveness of the proposed methods as well as single factor simulation analysis is verified based on data of Beijing subway Ba-Tong line.

2. Modelling of DC traction network
2.1 Train

The main circuit diagram for train is shown in Fig. 1 with three parts: traction system, auxiliary system, and braking resistor.

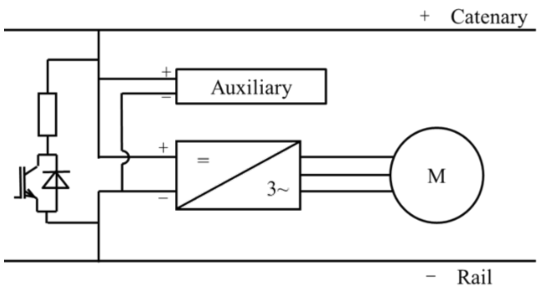


Fig. 1 Circuit diagram of train.

When power transmits between the catenary and wheel, some of it will be lost on pantograph, inverter, motor, and gears. Thus, there is a conversion coefficient between electric energy, input power, and kinetic energy, output power. When accelerating, the relationship between input power $P_{traction}$ (kW) and output power P_{motor} (kW) is shown in “Eq. (1)”

$$P_{traction}=\frac{P_{motor}}{\eta_{traction}} \quad (1)$$

Where $\eta_{traction}$ is the efficiency of electric drive system. Taking auxiliary power P_{aux} into consideration:

$$P_{train} = P_{traction} + P_{aux} \quad (2)$$

There are three states for trains while operating, named traction state, coasting state and braking state. Power flow from catenary to motor in traction state. In coasting state, no power flow in electrical traction system. The braking state of train can be divided into two situations: regenerative braking, in which kinetic power converted into electricity, and mechanical braking, in which kinetic energy is dissipated after being converted into heat energy by brake pads. In this study, all the braking is regenerative braking.

The train voltage, which is between catenary and rail at the train, will rise during regenerative braking. To ensure that the voltage does not exceed the safety upper limit and protect the device of train, when the voltage exceeds the maximum safe voltage, V_{over}, the braking resistor, R_{Bt}, will be put into use. In detail, if the voltage sensor sends an overvoltage signal to the brake resistance switch, the switch of the brake resistor, S_{Bt}, will be closed, then part of the regenerative braking current flows through the braking resistance and the train voltage will be reduced. This progress is also called "dynamic braking".

Train impedance is high according to the characteristics of power electronic components. So, the train can be considered as current source shown in "Fig. 2" and "Eq. (3)" (Tian et al., 2016). The unit of $I_{train}(t)$ is A, $V_{train}(t)$ is V, and $P_{train}(t)$ is kW.

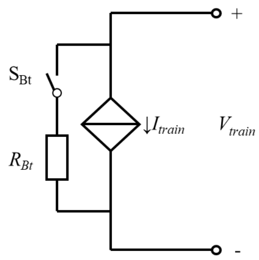


Fig. 2 Equivalent circuit of train.

$$I_{train}(t) = \frac{P_{train}(t)\times 10^3}{V_{train}(t)} \tag{3}$$

The power profile can be obtained by traction calculation. Several research study the energy-saving operation in single section train operation. Some algorithms, such as dynamic programming (Ichikawa et al., 2019), mixed integer linear programming (Wang et al., 2013) are used. Thus, the traction energy and regenerative energy of one section can be obtained.

2.2 Substation and traction network

The equivalent model of traction network is the basic of power flow calculation. The traction network of urban rail transit is composed of substation, catenary, and rail. The substation is the power source of the DC traction network. The catenary, which has resistance per length, transmit the power from substation to trains, regarded as the positive pole for trains. Trains are regarded as time-varying load when traction, and power source when regenerative braking. Rail is the negative pole of substation.

As for the substation, the core element of it is rectifier. Most of the substation use twelve-pulse rectifier of twenty-four-plus rectifier. Since the inner circuit of it is completive and full of non-linear elements, it is impossible to modelling it in linear functions. Hence, the Norton's theorem or Thevenin's theorem is used. However, the relationship of output voltage and current is not linear. In some cases, the piecewise linear model is used to describe the output current-voltage. In this study, the linear model is established due to it can reduce computation time. Besides, the ground braking resistance, which is used to reduce the voltage when the voltage between catenary and rail at the substation exceeds V_{over}. Thus, the substation has three states: traction state when the voltage of it less than output rated output voltage, which current flow from the substation to catenary; transition state when voltage higher than rated output voltage but less than maximum safe voltage, where no current flow in or out; braking state to keep the safety voltage, which the braking resistance is putted into use. The equivalent of substation is shown in "Fig. 3". V_{sub} is the output voltage of substation (V), R_{sub} is the inner resistance of rectifier (Ω), R_{Bs} is the ground braking resistance (Ω), and I_s is Norton's theorem current (A). During traction state, the switch S_{sub} connect to contact 1. During transition state, it does not connect to any contact. And during braking state, S_{sub} connect to contact 2.

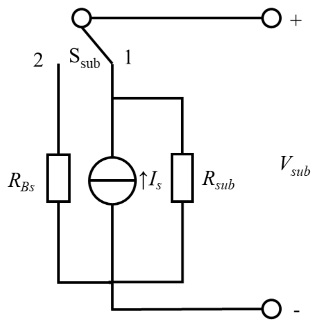


Fig. 3 Equivalent model of traction substation in Norton's theorem.

The DC traction network is shown in “Fig. 4”. Obviously, different sections are connected and the upwards and downwards are paralleled between substations.

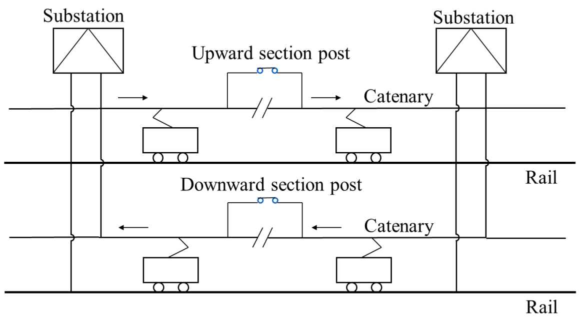


Fig. 4 DC traction network in one section.

2.3 Solution

Due to the non-linearization, time varying, and complexity topology of DC traction network, it is difficult to calculate by using linear functions. So, we can calculate the solution by iteration. In AC power grid or AC power traction network, the Newton-Raphson Method and Gauss-Seidel Method are widely used. Because the reactive power does not exist in ideal DC power systems, only the active power calculation in DC traction network is necessary. Hence, the current-vector method is constructed. However, there are resistance at substations and trains to keep the voltage in safety range. Thus, besides the normal iteration, it is necessary to check whether the state of substations and trains and voltage match.

Hence, the energy consumption W_{sub} can be written as “Eq. (4)”, where n is the number of substations, V_d is the rated output voltage of substation, and P_{sub} is the output power of substation, which is calculated by the product of output current, I_{sub_n}, and substation voltage, V_{sub_n}.

$$
\begin{aligned}
&W_{sub}=\sum_{n=1}^{sub} W_{sub\_n} \\
&W_{sub\_n}=\int P_{sub\_n}(t)\,\mathrm{d}t \\
&P_{sub\_n}(t)=\begin{cases} V_{sub\_n}(t)\cdot I_{sub\_n}(t), & V_{sub\_n}(t)<V_d \\ 0, V_{sub\_n}(t)>V_d \end{cases}
\end{aligned}
\tag{4}
$$

3. Timetable optimisation

3.1 Modelling

The energy flow of urban transit system is shown in "Fig. 5". It should be noted that only few of substation can invert energy to AC grid.

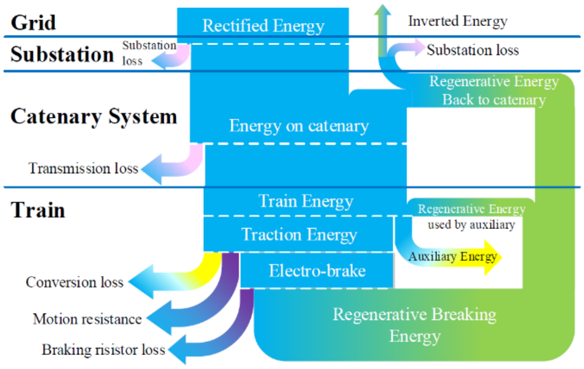


Fig. 5 Energy flow of urban transit system.

Obviously, some of the regenerative braking energy will be consumed by auxiliary systems. If others cannot be used by traction trains, it will be consumed by braking resistance at trains and substations, which is considered as loss. Hence, the improvement of regenerative energy utilisation will lead to energy saving. As for traction network, the train in the traction state absorbs energy from the catenary. If there are braking trains, the regenerative braking energy will be utilized by the train in the traction state. When the total regenerative braking energy generated is less than the total energy required for traction, the insufficient traction energy is provided by substations via catenary. When the electrical voltage exceeds the braking resistor operating voltage, the excess regenerative braking energy is dissipated by braking resistance.

Taking electricity supplied by substations as optimisation goal. Consider there are i trains running in multiple sections. Assume that all trains have the same performance of output, and all trains follow the same speed profile and same power profile in same section. The power of trains can be written as "Eq. 5", where t_{dep_n} is the departure time for train i at section n, $t_{traction\_n}$ is the end time of traction, t_{coast_n} is the end time of coasting, and t_{regen_n} is the end time of braking, which can be also considered as arriving time.

$$P_i(t, n)=\begin{cases} P_{traction\_i}(t, n),\ t\in[t_{dep\_n}, t_{traction\_n}) \\ 0,\ t\in[t_{traction\_n}, t_{coast\_n}) \\ P_{regen\_i}(t, n),\ t\in[t_{coast\_n}, t_{regen\_n}) \end{cases} \tag{5}$$

In this study, both section running time and dwell time are optimisation variables. Two optimization methods are designed, namely, adjusting only the stopping time (Method 1), and adjusting the interval running time and stopping time at the same time (Method 2). In method 1, the speed curve of trains remains unchanged. For the i-th train, the adjustment method is to shift its speed curve in the n-th section on the time axis by t_{i_n}.

$$P_i(t, n)=\begin{cases} P_{traction\_i}(t, n),\ t\in[t_{dep\_n}\pm t_{i\_n}, t_{traction\_n}\pm t_{i\_n}) \\ 0,\ t\in[t_{traction\_n}\pm t_{i\_n}, t_{aux\_n}\pm t_{i\_n}) \\ P_{regen\_i}(t, n),\ t\in[t_{aux\_n}\pm t_{i\_n}, t_{regen\_n}\pm t_{i\_n}) \end{cases} \tag{6}$$

In method 2, the stopping time of trains at each station and the section running time of each section may change, and the speed profile of trains will change at this time. The end time of generalized traction condition, the end time of coasting condition and the end time of regenerative braking condition of the n-th section are changed from $t_{traction\_n}$, t_{coast_n} and t_{regen_n} to $t'_{traction\_n}$, t'_{coast_n} and t'_{regen_n}. The required traction or regenerative braking power is changed from $P_{traction\_i}(t, n)$ and $P_{regen\_i}(t, n)$ to $P'_{traction\_i}(t, n)$ and $P'_{regen\_i}(t, n)$.

$$P_i'(t,n)=\begin{cases}P_{traction\_i}'(t,n),\ t\in[t_{dep\_n}'\pm t_{i\_n},\ t_{traction\_n}'\pm t_{i\_n})\\ 0,\ \mathrm{t}\in[t_{traction\_n}'\pm t_{i\_n},\ t_{aux\_n}'\pm t_{i\_n})\\ P_{regen\_i}'(t,n),\ t\in[t_{aux\_n}'\pm t_{i\_n},\ t_{regen\_n}'\pm t_{i\_n})\end{cases} \tag{7}$$

There are four constraints of timetable optimisation. First, the headway of trains $t_{headway}$ cannot be shortened too much for keeping safety.

$$t_{headway} > t_{headway\_min} \tag{8}$$

On the one hand, passengers need time to get on or off the train, which can be considered as the minimum dwell time. On the other hand, if train stop at a station too much time, passengers will feel impatient. Hence, the dwell time t_{dwell_n} can only be adjusted in a certain range.

$$t_{dwell\_n\_min} < t_{dwell\_n} < t_{dwell\_n\_max} \tag{9}$$

When adjusting section running time $t_{section\_n}$, it is necessary to prevent the train from running at the maximum capacity and unable to run into the given section running time, but also to ensure the transportation efficiency and avoid a train running too much time in a certain section.

$$\begin{gathered}\sum_{n=1}^{section}(t_{traction\_n}+t_{aux\_n}+t_{regen\_n})\\ t_{section\_n\_min}<t_{section\_n}<t_{section\_n\_max}\end{gathered} \tag{10}$$

To avoid rearranging the using table, the total running time t_{total} is not adjusted.

$$t_{total}'=t_{total} \tag{11}$$

Due to the computation time, in optimisation model, the non-linear of DC traction network system, braking resistance are not considered. Only take the power, which also means energy generated or needed by train into consideration. Hence, the optimisation goal of timetable optimisation can be written as

$$\begin{aligned}&\min W_{sub}=\sum_{n=1}^{sub}W_{sub\_n}\\ &\text{s.t.}\ \ t_{section\_n}=\sum_{n=1}^{sub}W_{sub\_n}\\ &\qquad t_{section\_n\_min}<t_{section\_n}<t_{section\_n\_max}\\ &\qquad t_{dwell\_n\_min}<t_{dwell\_n}<t_{dwell\_n\_max}\\ &\qquad t_{headway}>t_{headway\_\mathrm{max}}\\ &\qquad t_{total}'=t_{total}\end{aligned} \tag{12}$$

3.2 Solution approach

One of the power profiles is shown in “Fig. 6”.

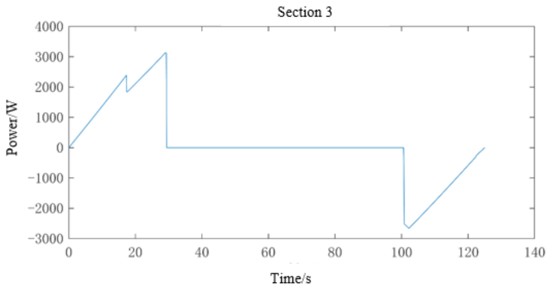


Fig. 6 Train power profile in Section 3.

Due to the no smooth power profile, traditional mathematical optimisation method is not fit in this study. Besides, the large-scale variables and complex constraints of optimisation model also leads to use heuristic algorithms. Since the genetic algorithm is used in several railway-related optimisation problems (He et al., 2020), this study designs an improved genetic algorithm (GA) to solve this problem.

In traditional genetic algorithm, each individual only has one chromosome, which is encoded. However, the uplink and downlink of DC traction network are connected in parallel with the bilateral power supply, and the upwards and downwards trains are independent, except before the first station and after the terminal, of each other. Referring to polyploid organisms in biology, an individual includes two different chromosomes, represent the timetable of upward and downward respectively, which is the improvement of traditional genetic algorithm. In detail, gene is the timetable of train i, include all the departure time and arrive time at each station for one ward. And chromosome is the timetable of train in one ward. Each individual is made for two chromosomes. This is shown schematically in “Fig. 7”.

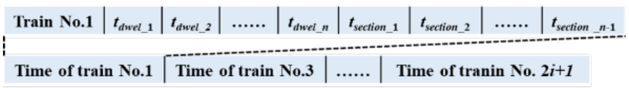


Fig. 7 Schematic diagram of a gene and chromosome.

Each individual is made by traversing the upward and downward chromosomes.

For the improved genetic algorithm, the coding method is natural number coding. The crossover in this study is changing the timetable of one or several trains in different chromosome as shown in “Fig. 8”.

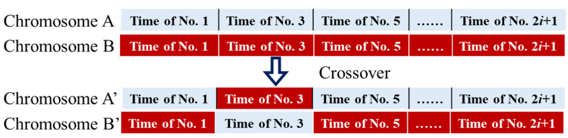


Fig. 8 Crossover method.

Due to the constraints “Eq. 8 – 11”, the adjustment amount of interval running time is integer and bounded, and the sum of all adjustments for train i is 0 as shown in “Eq. 13”.

$$\sum t_{i\_n} = 0 \tag{13}$$

Taking energy consumption of each individual as fitness function. To accelerate the computing progress, the calculation of energy consumption is simplified as direct summation of power, including traction, braking and auxiliary power, and integration of time, as shown in “Eq. 14”.

$$W_{sub}= \int P_{sub}(t)\mathrm{d}t$$

$$P_{sub}(t)= \begin{cases} \sum_{i=1}^{train} P_i(t),\ P_i(t) > 0 \\ 0,\ \sum_{i=1}^{train} P_i(t) < 0 \end{cases} \tag{14}$$

The purpose of selection is to ensure that the genes of excellent individuals can be inherited to the offspring and exclude the inferior genes. The genes inherited to the offspring should ensure both high quality and diversity. Therefore, a hybrid strategy based on RWS and optimal selection is adopted as the selection method.

4. Case study

4.1 Parameter setting

Beijing subway Ba-Tong line is an urban rail line, which connect the CBD and administrative sub centre. It has 23.4km length and 14 stations (only 13 substations are considered). The parameter of Ba-Tong line shown in Table 1

Table 1 Parameters of Beijing subway Ba-Tong line.

Name	Symbol	Value	Unit
Substation output rated voltage	V_d	825	V
Substation maximum voltage	V_{over}	900	V
Inner resistance of rectifier	R_{sub}	0.02	Ω
Substation ground braking resistance	R_{bt}	1.2	Ω
Catenary resistance per unit length	r_3	0.0081	Ω/km
Rail resistance per unit length	r_r	0.0136	Ω/km

The auxiliary power is not constant during the whole year. Due to air condition, the auxiliary power in summer and winter is higher than spring and autumn. In this study, the auxiliary power is taken into consideration.

In this part, the optimisation model is used to optimise the timetable. And the calculation model, used the DC traction network to calculate the actual energy consumption from the power system. To prove the energy consumption from DC traction network is necessary, this study also make a comparation with traditional energy consumption calculation method, which means without calculation of DC traction method. The different optimisation method of cases is shown in Table 2. The auxiliary power is taken into consideration during optimising or energy consumption calculation in some cases. And the parameters are shown in Table 3.

Table 2 Different optimisation methods.

Timetable optimisation method	P_{aux} when optimising	P_{aux} when calculate energy consumption	Case
Change dwell time only	×	×	A
Change dwell & section running time	×	×	B
Change dwell time only	×	○	C
Change dwell & section running time	×	○	D
Change dwell time only	○	○	E

Table 3 Parameters in GA.

RWS rate	Mutation probability	Crossover probability	Population size	Generation
0.8	0.4	0.8	625	1500

4.2 Case A & Case B

In these cases, suggest the auxiliary power is 0 kW. The energy consumption and comparison are shown in Table 4. "Energy saving" means that how much energy of the optimised timetable is reduced compared with the original energy consumption in same calculation method. "Without calculation of DC traction network" means the energy consumption is calculated by direct summation of instantaneous power, as shown in "Eq. 14". "Within solution of DC traction network" means the energy consumption is calculated by the method mentioned in section 2.

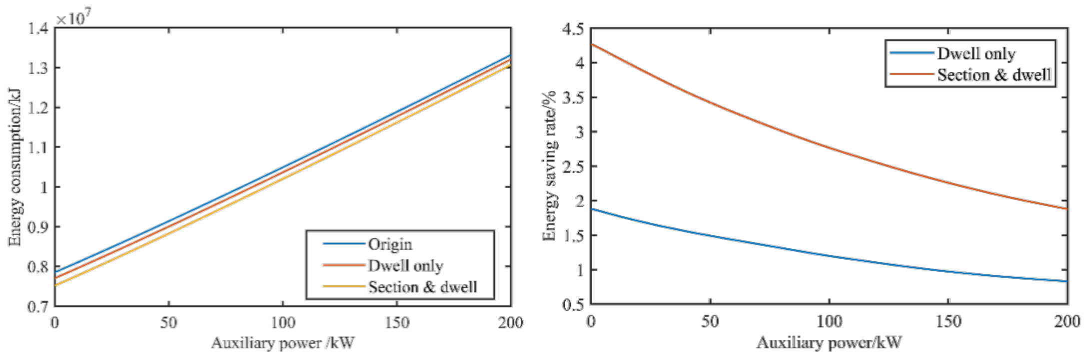
Table 4 Energy consumption comparation in case A & B.

Case	Without calculation of DC traction network			Within solution of DC traction network		
	Energy consumption [MJ]	Energy saving [MJ]	Energy saving rate	Energy consumption [MJ]	Energy saving [MJ]	Energy saving rate
Origin	5788.0	N/A	N/A	7852.9	N/A	N/A
A	5556.8	231.2MJ	4.00%	7704.9	147.7MJ	1.88%
B	5548.2	239.8MJ	4.14%	7517.4	335.4MJ	4.27%

Because the over voltage protection in traction network, some regenerative energy is consumed by train braking resistor or ground braking resistance. So, not all the regenerative energy can be used by accelerating trains, which leads that energy consumption within DC traction network is higher than without calculation of DC traction network. Hence, the accurate energy consumption calculation is important.

4.3 Case C & Case D

In these cases, different auxiliary power is taken into consideration in calculation model. Assume that the auxiliary power changes gradually from 0 to 200kW. The energy consumption with auxiliary power and energy saving rate with auxiliary power shown in "Fig. 9 (a)" and "Fig. 9(b)".



(a) (b)

Fig. 9 Energy with auxiliary power

When the auxiliary power is 100kW, the energy consumption and saving rate is shown in Table 5.

Table 5 Energy consumption when P_{aux} = 100kW.

Case	Energy consumption [MJ]	Energy saving [MJ]	Energy saving rate
Origin	10490.2	N/A	N/A
C	10364.5	125.7	1.20%
D	10200.0	290.2	2.77%

Hence, auxiliary power will significantly reduce the energy saving rate.

4.4 Case E

From the conclusion above, taking auxiliary power into consideration in optimisation model. Suggest the auxiliary power of trains is 100kW. The energy consumption comparation with Case C is shown in Table 6.

Table 6 Energy consumption comparation.

Result	Energy consumption [MJ]	Energy saving [MJ]	Energy saving rate
Origin	10490.2	N/A	N/A
C	10364.5	125.7	1.20%
E	9758.1	732.1	6.98%

In Case E, compared with Case C, although the train in coasting state, it also absorbs power from catenary for auxiliary system. So, besides overlapping traction and regenerative braking, optimisation model will also tend to synchronise coasting and regenerative braking. Hence, considering auxiliary power in optimisation leads to saves more energy.

5. Conclusions

The DC traction network can take train braking resistor and ground braking resistance into consideration. Within its calculation, the power flow and traction network consumption can be shown in results. So, the importance of accurate energy consumption is proved. In addition, by single factor simulation analysis, this paper verifies the effect of optimized timetables when changing speed profile, which leads to improve the energy efficiency or regenerative energy utilisation. The increase of auxiliary power of trains will consume more energy and decrease the energy-saving efficient, but taking it into consideration in optimisation model leads to saves more energy.

References

Alnuman, H., Gladwin, D. and Foster, M. (2018), “Electrical modelling of a DC railway system with multiple trains”, Energies, Vol. 11, No. 11, DOI: 10.3390/en11113211.

Cai, Y., Irving, M.R. and Case, S.H. (1995a), “Modelling and numerical solution of multibranched DC rail traction power systems”, IEE Proceedings: Electric Power Applications, Vol. 142, No. 5, pp. 323–328.

Cai, Y., Irving, M.R. and Case, S.H. (1995b), “Iterative techniques for the solution of complex DC-rail-traction systems including regenerative braking”, IEE Proceedings - Generation, Transmission and Distribution, Vol. 142, No. September, pp. 445–452.

He, D., Yang, Y., Chen, Y., Deng, J., Shan, S., Liu, J. and Li, X. (2020), “An integrated optimization model of metro energy consumption based on regenerative energy and passenger transfer”, Applied Energy, Elsevier Ltd, Vol. 264, DOI: 10.1016/j.apenergy.2020.114770.

Ichikawa, S., and Miyatake, M. (2019), “Energy Efficient Train Trajectory in the Railway System with Moving Block Signaling Scheme”, IEEJ Journal of Industry Applications, Vol. 8, Issue 4, pp.586–591.

Jin, B., Sun, P., Wang, Q. and Feng, X. (2020), “Two-step method to reduce metro transit energy consumption by optimising speed profile and timetable”, IET Intelligent Transport Systems, Vol. 14, No. 9, pp. 1097–1107.

Ku, B. and Liu, J. (2020), “Modelling and numerical solution of ungrounded DC rail traction system load flow using a ladder circuit – part I: circuit modelling”, IET Generation, Transmission & Distribution, Vol. 14, No. 1, pp. 1–10.

Li, X. and Yang, X. (2020), Subway Energy-Efficient Management, Springer Singapore, DOI: 10.1007/978-981-15-7785-7.

Lv, H., Zhang, Y., Huang, K., Yu, X. and Wu, J. (2019), “An energy-efficient timetable optimization approach in a bi-directionurban rail transit line: A mixed-integer linear programming model”, Energies, Vol. 12, No. 14, pp. 1–24.

Mellitt, B., Goodman, C.J. and Arthurton, R.I.M. (1978), “Simulator for studying operational and power-supply conditions in rapid-transit railways”, Proceedings of the Institution of Electrical Engineers, Vol. 125, No. 4, p. 298.

Takeuchi, Y,. Ogawa, T., Sato, K., Morimoto, H. and Saito, T. (2019), “Power Control Optimization of an Energy Storage System in DC Electric Railways”, IEEJ Journal of Industry Applications, Vol. 8, Issue 5, pp.827-834.

Tian, Z., Hillmansen, S., Roberts, C., Weston, P., Zhao, N., Chen, L. and Chen, M. (2016), “Energy evaluation of the power network of a DC railway system with regenerating trains”, IET Electrical Systems in Transportation, Vol. 6, No. 2, pp. 41–49.

Wang, Y., De Schutter, B., van den Boom, T.J.J. and Ning, B. (2013), “Optimal trajectory planning for trains - A pseudospectral method and a mixed integer linear programming approach”, Transportation Research Part C: Emerging Technologies, Vol. 29, pp. 97–114.

Wijaya, F.P., Watanabe, S., Saito, T., Kobayashi, H. and Kondo, K. (2017), “Advantages and technical issues of regenerative brake method at all over the speed range”, Mechanical Engineering Journal, Vol. 4, Issue 1, p. 16-00473

Yang, X., Li, X., Ning, B. and Tang, T. (2016), “A survey on energy-efficient train operation for urban rail transit”, IEEE Transactions on Intelligent Transportation Systems, Vol. 17, No. 1, pp. 2–13.

Yang, X., Wu, J., Sun, H., Gao, Z., Yin, H. and Qu, Y. (2019), “Performance improvement of energy consumption, passenger time and robustness in metro systems: A multi-objective timetable optimization approach”, Computers and Industrial Engineering, Elsevier, Vol. 137, No. September, p. 106076.